Bull. Cal. Math. Soc., **103,** (5) 403–412 (2011)

APPLICATION OF BAILEY'S PAIR TO q-IDENTITIES

REMY Y. DENIS[1], S.N. SINGH[2], S.P. SINGH[2] AND NIDHI SAHNI[1]

(*Received 5 October 2010*)

Abstract : Bailey's (1949) lemma has been a very useful tool to establish q-series identities. Various mathematicians have exploited this result to arrive at Rogers-Ramanujan as well as q-series identities. Slater (1952) was one of the pionner workers who establish more than a hundred identities. Andrews (1970, 1984, 1987 and 2005) has given a very valuable account of its applications, using certain pairs of sequences, called Bailey's Pairs. In this paper we shall use some of known Bailey's pairs to establish certain interesting q-series identities of special nature.

1. Introduction. For $|q| < 1$ and α, real or complex, let

$$[\alpha; q]_n = (1-\alpha)(1-\alpha q)(1-\alpha q^2 \cdots (1-\alpha q^{n-1}) \tag{1.1}$$

when $n \geq 1$ and $[\alpha; q]_0 = 1$.

Also,

$$[\alpha; q]_\infty = \prod_{n=0}^{\infty}(1-\alpha q^n)$$

and

$$[a_1, a_2, \cdots, a_r; q]_n = [a_1; q]_n [a_2; q]_n \cdots [a_r; q]_n. \tag{1.2}$$

Now, we define a basic hypergeometric function,

$${}_r\phi_s\left[\begin{matrix} a_1, a_2, \cdots, a_r; q; z \\ b_1, b_2, \cdots, b_s; q^\lambda \end{matrix}\right] = \sum_{n=0}^{\infty} \frac{[a_1, a_2, \cdots, a_r; q]_n z^n q^{\lambda n(n-1)/2}}{[q, b_1, b_2, \cdots, b_s; q]_n} \tag{1.3}$$

convergent for $|z| < \infty$ when $\lambda > 0$ and for $|z| < 1$ when $\lambda = 0$.

Basic hypergeometric functions play fundamental role in q-series identities leading to partition theory and combinatorial identities. The most important ones are the Rogers-Ramanujan's identities. For further details one refer to Slater (1952), Carlitz (1959), Andrews (1970, 1984 and 1987), Garrett (2006), Garrett, Ismail and Stanton (1999).

With the help of Bailey's pairs we have evaluated certain interesting q-identities. Bailey (1949) gave the following very useful transformation.

If

$$\beta_n = \sum_{r=0}^{n} \alpha_r u_{n-r} v_{n+r} \text{ and } \gamma_n = \sum_{r=0}^{\infty} \delta_{r+n} u_r v_{r+2n} \tag{1.4}$$

then, under suitable convergence conditions,

$$\sum_{n=0}^{\infty} \alpha_n \gamma_n = \sum_{n=0}^{\infty} \beta_n \delta_n, \tag{1.5}$$

where α_r, β_r, u_r and v_r are functions of r alone.

Again, let α_r be a sequence of complex numbers and

$$\beta_n = \sum_{r=0}^{n} \frac{\alpha_r}{[q;q]_{n-r}[aq;q]_{n+r}}, \tag{1.6}$$

then the sequences $\{\alpha_n\}$ and $\{\beta_n\}$ are said to form a Bailey's pair that satisfies (2.1.6). We shall use certain Bailey's pairs to establish some q-series identities of special nature.

2. Results used. We shall make use of the following Bailey's pairs (say) α_n and β_n;

(i)
$$\alpha_n = \frac{(1-aq^{2n})[\alpha;q]_n(-)^n q^{n(n-1)/2}}{(1-a)[q;q]_n}, \quad n \geq 0 \tag{2.1}$$

$$\beta_n = 1 \text{ when } n=0 \text{ and } \beta_n = 0 \text{ if } n > 0 \tag{2.2}$$

(cf. Stanton [1989; page 2])

(ii)
$$\alpha_n = \begin{cases} 1; & n=0 \\ (-)^n(z^n q^{n(n-1)/2} + z^{-1}q^{n(n+1)/2}); & n \geq 1. \end{cases} \tag{2.3}$$

$$\beta_n = \frac{[z, q/z;q]_n}{[q;q]_{2n}} \text{ for } n \geq 0. \tag{2.4}$$

(cf. Berndt [1994; 23.4, p.159])

(iii)
$$\alpha_n = \begin{cases} 1; & n = 0 \\ (-)^n(1-aq^{2n})[aq;q]_{n-1}a^nq^{n(3n-1)/2}/[q;q]_n; & n \geq 1. \end{cases} \tag{2.5}$$

and
$$\beta_n = \frac{1}{[q;q]_n}. \tag{2.6}$$

(cf. Verma [1980; (i) p.771])

(iv)
$$\alpha_0 = 1, \quad \alpha_n = (1-aq^{2n})\frac{[aq;q]_{n-1}[b;q]_n a^n q^{n^2}}{[q;q]_n[aq/b;q]_n b^n}, \quad n \geq 1 \tag{2.7}$$

and
$$\beta_n = \frac{[-aq^{n+1}/b;q]_n}{[q^2;q^2]_n[-aq;q]_{2n}[aq/b;q]_n}. \tag{2.8}$$

(cf. Verma [1980; (iii) p.772])

(v)
$$\alpha_0 = 1, \quad \alpha_n = \frac{(-a)^n(1-aq^{2n})[aq;q]_{n-1}q^{n(3n-1)/2}}{[q;q]_n} \quad \text{for } n \geq 1 \tag{2.9}$$

and
$$\beta_n = \frac{[aq;q]_{3n}}{[q^3;q^3]_n[a^3q^3;q^3]_{2n}} \quad \text{for } n \geq 0 \tag{2.10}$$

(cf. Verma [1980; (iv) p.772])

3. Main Results. In this section we shall establish our results.

If we take
$$\delta_r = [\rho_1, \rho_2; q]_r (aq/\rho_1\rho_2)^r$$
in (2.1) together with
$$u_r = \frac{1}{[q;q]_r}; \quad v_r = \frac{1}{[aq;q]_r},$$
then
$$\begin{aligned}\gamma_n &= \frac{1}{[aq;q]_{2n}} \sum_{r=0}^{\infty} \frac{[\rho_1,\rho_2;q]_{r+n}(aq/\rho_1\rho_2)^{r+n}}{[q;q]_r[aq^{2n+1};q]_r} \\ &= \frac{[\rho_1,\rho_2;q]_n(aq/\rho_1\rho_2)^n}{[aq;q]_{2n}} \, {}_2\phi_1\left[\begin{matrix}\rho_1 q^n, \rho_2 q^n; q; aq/\rho_1\rho_2 \\ aq^{2n+1}\end{matrix}\right]\end{aligned} \tag{3.1}$$

Now, evaluating ${}_2\phi_1$ on the right side of (3.1), we get from (1.4) that if
$$\beta_n = \sum_{r=0}^{\infty} \frac{\alpha_r}{[q;q]_{n-r}[aq;q]_{n+r}}, \tag{3.2}$$

then

$$\sum_{n=0}^{\infty}[\rho_1,\rho_2;q]_n(aq/\rho_1\rho_2)^n\beta_n = \frac{[aq/\rho_1,aq/\rho_2;q]_\infty}{[aq,aq/\rho_1\rho_2;q]_\infty} \times \sum_{n=0}^{\infty}\frac{[\rho_1,\rho_2;q]_n(aq/\rho_1\rho_2)^n\alpha_n}{[aq/\rho_1,aq/\rho_2;q]_n} \tag{3.3}$$

The sequences $\{\alpha_r\}$ and $\{\beta_r\}$, given by (3.3) are known as Bailey's pair, which leads to a large number of Rogers-Ramanujan type of identities. It was Bailey who pointed out that if the following summation

$$ {}_2\phi_1\left[\begin{matrix}a,b;q;c/ab\\ cq\end{matrix}\right] = \frac{[cq/a,q/b;q]_\infty}{[cq,cq/ab]_\infty}\times\left\{\frac{ab(1+c)-c(a+b)}{ab-c}\right\} \tag{3.4}$$

is involved instead of Gauss' theorem, then (1.4), (1.5) and (3.1) along with

$$\delta_r[\rho_1,\rho_2;q]_r(a/\rho_1\rho_2)^r$$

will lead to the following:

If

$$\beta_n=\sum_{r=0}^{n}\frac{\alpha_r}{[q;q]_{n-r}[aq;q]_{n+r}}, \tag{3.5}$$

then

$$\sum_{n=0}^{\infty}[\rho_1,\rho_2;q]_n(a/\rho_1\rho_2)^n\beta_n = \frac{[aq/\rho_1;aq/\rho_2;q]_\infty}{[aq,aq/\rho_1\rho_2;q]_\infty} \times\sum_{n=0}^{\infty}\frac{[\rho_1,\rho_2;q]_n(a/\rho_1\rho_2)^n}{[aq/\rho_1,aq/\rho_2;q]_n}\left[\frac{\rho_1\rho_2(1+aq^{2n})-aq^n(\rho_1+\rho_2)}{\rho_1\rho_2-a}\right]=\alpha_n, \tag{3.6}$$

which will reduces to Rogers-Ramanujan type of identities of a different nature.

Motivated by the above observation, we attempt to investigate these identities.

If we make use of (2.1) and (2.2) in (3.6) we get

$$\sum_{n=0}^{\infty}\frac{[\rho_1,\rho_2;q]_n(a/\rho_1\rho_2)^n}{[aq/\rho_1,aq/\rho_2;q]_n}\left[\frac{\rho_1\rho_2(1+aq^{2n})-aq^n(\rho_1+\rho_2)}{(\rho_1\rho_2-a)}\right] \frac{(-)^n[a;q]_n(1-aq^{2n})q^{n(n-1)/2}}{(1-a)[q;q]_n} = \frac{[aq,aq/\rho_1\rho_2;q]\infty}{[aq/\rho_1,aq/\rho_2';q]_\infty}. \tag{3.7}$$

As $\rho_1, \rho_2 \to \infty$ in (3.7), we get

$$\sum_{n=0}^{\infty} \frac{q^{3n(n-1)/2} a^n (1 - a^2 q^{4n})(-1)^n [a;q]_n}{(1-a)[q;q]_n} = [aq;q]_\infty \tag{3.8}$$

Letting $a \to 1$ and after separating the term corresponding to $n = 0$ in (3.8), we get

$$2 + \sum_{n=1}^{\infty} (-)^n q^{3n(n-1)/2} (1 + q^{2n})(1 + q^n) = [q;q]_\infty \tag{3.9}$$

If we set $a \to -1$ in (3.8), we get

$$\sum_{n=0}^{\infty} \frac{q^{3n(n-1)/2}(1 - q^{4n})[-q;q]_{n-1}}{[q;q]_n} = [-q;q]_\infty \tag{3.10}$$

Further, for $a = q$ in (3.8), we get

$$\sum_{n=0}^{\infty} (-)^n q^{n(3n-1)/2} - q^2 \sum_{n=0}^{\infty} (-)^n q^{n(3n+7)/2} = [q;q]_\infty \tag{3.11}$$

Further, making use of (2.3) and (2.4) in (3.6), we get, by replacing a by 1.

$$\begin{aligned}\sum_{n=0}^{\infty} \frac{[\rho_1, \rho_2, z, q/z;q]_n (\rho_1\rho_2)^{-n}}{[q;q]_{2n}} &= \frac{[q/\rho_1, q/\rho_2;q]_\infty}{[q, q/\rho_1\rho_2;q]_\infty} \Bigg[\sum_{n=1}^{\infty} \frac{[\rho_1, \rho_2;q]_n (-\rho_1\rho_2)^{-n}}{[q/\rho_1, q/\rho_2;q]_n} \\ &\times \left\{ \frac{\rho_1\rho_2(1+q^{2n}) - q^n(\rho_1+\rho_2)}{\rho_1\rho_2 - 1} \right\} \left\{ z^n q^{n(n-1)/2} + z^{-n} q^{n(n+1)/2} \right\} \\ &+ \frac{2\rho_1\rho_2 - (\rho_1+\rho_2)}{\rho_1\rho_2 - 1} \Bigg].\end{aligned} \tag{3.12}$$

As ρ_1 and $\rho_2 \to \infty$ in (3.12), after some simplification, we get

$$\begin{aligned}&\sum_{n=0}^{\infty} \frac{[z, q/z;q]_n q^{n(n-1)}}{[q;q]_{2n}} \\ &= \frac{1}{[q;q]_\infty} \left\{ \sum_{n=-\infty}^{\infty} (-n)^n z^n q^{3n(n-1)/2} + \sum_{n=-\infty}^{\infty} (-)^n z^n q^{n(3n+1)/2} \right\}.\end{aligned} \tag{3.13}$$

Now, evaluating the bilateral series on the right hand side of (3.13) with the help of Jacobi's triple product identity, it yields

$$\sum_{n=0}^{\infty}\frac{[z,q/z;q]_n q^{n(n-1)}}{[q;q]_{2n}}=\frac{[q^3;q^3]_\infty}{[q;q]_\infty}\left\{[z,q^3/z;q^3]_\infty+[zq^2,q/z;q^3]_\infty\right\}. \tag{3.14}$$

Now, if we put $z=-1$ in (3.14), we get

$$\sum_{n=0}^{\infty}\frac{[-1,-q;q]_n q^{n(n-1)}}{[q;q]_{2n}}=\frac{[q^3;q^3]_\infty}{[q;q]_\infty}\left\{2[-q^3;q^3]^2_\infty+[-q,-q^2;q^3]_\infty\right\}. \tag{3.15}$$

Now, invoking (2.5), (2.6) in (3.6) yields

$$\sum_{n=0}^{\infty}\frac{[\rho_1,\rho_2;q]_n(a/\rho_1\rho_2)^n}{[q;q]_n}=\frac{[aq/\rho_1,aq/\rho_2;q]_\infty}{[aq,aq/\rho_1\rho_2;q]_\infty}\left[\left\{\frac{\rho_1\rho_2(1+a)-a(\rho_1+\rho_2)}{\rho_1\rho_2-a}\right\}\right.$$
$$+\sum_{n=1}^{\infty}\frac{[\rho_1,\rho_2;q]_n(a/\rho_1\rho_2)^n}{[aq/\rho_1,aq/\rho_2;q]_n}\left\{\frac{\rho_1\rho_2(1+aq^{2n})-aq^n(\rho_1+\rho_2)}{\rho_1\rho_2-a}\right\}$$
$$\left.\times\frac{(-1)^n(1-aq^{2n})[aq;q]_n a^n q^{n(3n-1)/2}}{(1-aq^n)[q;q]_n}\right]. \tag{3.16}$$

If ρ_1 and $\rho_2\to\infty$ in (3.16), and a is replaced by q, then

$$\sum_{n=0}^{\infty}\frac{q^{n^2}}{[q;q]_n}=\frac{1}{[q;q]_\infty}\sum_{n=0}^{\infty}(-)^n q^{n(5n-1)/2}(1-q^{4n+2}). \tag{3.17}$$

Since

$$\sum_{n=0}^{\infty}\frac{q^{n^2}}{[q;q]_n}=\frac{1}{[q;q^5]_\infty[q^4;q^5]_\infty},$$

we get, from (3.17)

$$\sum_{n=0}^{\infty}(-)^n q^{n(5n+1)/2}(1-q^{4n+2})=[q^2,q^3;q^5,q^5]_\infty. \tag{3.18}$$

If $\rho_1,\rho_2\to\infty$ in (3.16), and a is replaced by 1, we get

$$\sum_{n=0}^{\infty}\frac{q^{n(n-1)}}{[q;q]_n}=\frac{1}{[q;q]_\infty}\left\{2+\sum_{n=1}^{\infty}(-)^n q^{n(5n-3)/2}(1+q^n)(1+q^{2n})\right\}$$
$$=\frac{1}{[q;q]_\infty}\left\{\sum_{n=-\infty}^{\infty}(-)^n q^{n(5n-3)/2}+\sum_{n=-\infty}^{\infty}(-n)^n q^{n(5n-1)/2}\right\},$$

which reduces to

$$\sum_{n=0}^{\infty} \frac{q^{n(n-1)}}{[q;q]_n} = \frac{1}{[q^2,q^3;q^5]_\infty} + \frac{1}{[q,q^4,q^5]_\infty}. \tag{3.19}$$

The above identity can also be obtained with the help of the two celebrated Rogers-Ramanaujan identities. It may be noted that (3.19) is a special case of a result due to Cariltz [1959], with $k = 1$.

Invoking (2.7), (2.8) and (3.6), we get

$$\sum_{n=0}^{\infty} \frac{[\rho_1,\rho_2;q]_n (a/\rho_1\rho_2)^n [-aq^{n+1}/b;q]_n}{[q^2;q^2]_n[-aq;q]_{2n}[aq/b;q]_n} = \frac{[aq/\rho_1, aq/\rho_2;q]_\infty}{[aq, aq/\rho_1\rho_2;q]_\infty}$$

$$\times \left[\frac{\rho_1\rho_2(1+a) - a(\rho_1+\rho_2)}{\rho_1\rho_2 - a} + \sum_{n=1}^{\infty} \frac{[\rho_1,\rho_2;q]_n (a/\rho_1+\rho_2)^n}{[aq/\rho_1, aq/\rho_2;q]_n} \right.$$

$$\left. \left\{ \frac{\rho_1\rho_2(1+aq^{2n}) - aq^n(\rho_1+\rho_2)}{\rho_1\rho_2 - a} \right\} \times \frac{(1-aq^{2n})[aq;q]_{n-1}[b;q]_n a^n q^{n^2}}{[q;q]_n[aq/b;q]_n b^n} \right] \tag{3.20}$$

As $\rho_1, \rho_2 \to \infty$, (3.20) yields

$$\sum_{n=0}^{\infty} \frac{q^{n(n-1)} a^n [-aq^{n+1}/b;q]_n}{[q^2;q^2]_n[-aq;q]_{2n}[aq/b;q]_n}$$

$$= \frac{1}{[aq;q]_\infty} \left[1 + a + \sum_{n=1}^{\infty} \frac{a^{2n} q^{n(2n-1)} (1+aq^{2n})(1-aq^{2n})[aq;q]_n[b;q]_n}{(1-aq^n)[q, aq/b;q]_n b^n} \right]. \tag{3.21}$$

Now, taking $a = 1$ in (3.21), we get,

$$\sum_{n=0}^{\infty} \frac{q^{n(n-1)} [-q^{n+1}/b;q]_n}{[q^2;q^2]_n[-q;q]_{2n}[q/b;q]_n}$$

$$= \frac{1}{[q;q]_\infty} \left[2 + \sum_{n=1}^{\infty} \frac{q^{n(2n-1)}(1+q^{2n})(1+q^n)[b;q]_n}{[q/b;q]_n b^n} \right]. \tag{3.22}$$

The same can be written as

$$\sum_{n=0}^{\infty} \frac{q^{n(n-1)} [-q^{n+1}/b;q]_n}{[q^2;q^2]_n[-q;q]_{2n}[q/b;q]_n}$$

$$= \frac{1}{[q;q]_\infty} \left[2 + \sum_{n=1}^{\infty} \frac{(1+q^n) q^{2n^2-n}[b;q]_n}{[q/b;q]_n b^n} + \sum_{n=1}^{\infty} \frac{(1+q^n) q^{2n^2+n}[b;q]_n}{[q/b;q]_n b^n} \right]$$

$$= \frac{1}{[q;q]_\infty}\left[2+\sum_{n=1}^{\infty}\frac{(1+q^n)q^{2n^2-n}[b;q]_n}{[q/b;q]_n b^n}+\sum_{n=-1}^{-\infty}\frac{(1+q^n)q^{2n^2-n}[b;q]_n}{[q/b;q]_n b^n}\right]$$

$$= \frac{1}{[q;q]_\infty}\left\{\sum_{n=-\infty}^{\infty}\frac{(1+q^n)q^{2n^2-n}[b;q]_n}{[q/b;q]_n b^n}\right\}$$

As $b \to \infty$, the above relation yields,

$$\sum_{n=0}^{\infty}\frac{q^{n(n-1)}}{[q^2;q^2]_n[-q;q]_{2n}} = \frac{1}{[q;q1]_\infty}\left\{\sum_{-\infty}^{\infty}(-)^n q^{n(5n-3)/2}+\sum_{-\infty}^{\infty}(-)^n q^{n(5n-1)/2}\right\} \tag{3.23}$$

Jacobi's triple product identity is employed to obtain,

$$\sum_{n=0}^{\infty}\frac{q^{n(n-1)}}{[q^2;q^2]_n[-q;q]_{2n}} = \frac{1}{[q^2,q^3;q^5]_\infty}+\frac{1}{[q,q^4;q^5]_\infty}. \tag{3.24}$$

Finally, using (2.9) and (2.10) in (3.6), we get

$$\begin{aligned}
&\sum_{n=0}^{\infty}\frac{[\rho_1,\rho_2;q]_n(a/\rho_1\rho_2)^n[aq;q]_{3n}}{[q^3;q^3]_n[a^3q^3;q^3]_{2n}} \\
&= \frac{[aq/\rho_1,aq/\rho_2;q]_\infty}{[aq,aq/\rho_1\rho_2;q]_\infty}\left[\frac{\rho_1\rho_2(1+a)-a(\rho_1+\rho_2)}{\rho_1\rho_2-a}\right. \\
&\quad +\sum_{n=1}^{\infty}\frac{[\rho_1,\rho_2;q]_n(a/\rho_1\rho_2)^n}{[aq/\rho_1,aq/\rho_2;q]_n}\left\{\frac{\rho_1\rho_2(1+aq^{2n})-aq^n(\rho_1+\rho_2)}{\rho_1\rho_2-a}\right\} \\
&\quad \left.\times\frac{(-a)^n(1-aq^{2n})[aq;q]_n q^{n(3n-1)/2}}{[q;q]_n(1-aq^n)}\right]
\end{aligned} \tag{3.25}$$

Now, letting $\rho_1,\rho_2 \to \infty$ in (3.25), we get

$$\begin{aligned}
&\sum_{n=0}^{\infty}\frac{a^n q^{n(n-1)}[aq;q]_{3n}}{[q^3,q^3]_n[a^3q^3;q^3]_{2n}} \\
&= \frac{1}{[aq;q]_\infty}\left\{(1+a)+\sum_{n=1}^{\infty}\frac{(-)^n a^{2n}q^{n(5n-3)/2}(1-a^2q^{4n})[aq;q]_n}{(1-aq^n)[q;q]_n}\right\}
\end{aligned} \tag{3.26}$$

and letting $a \to 1$ in the above, we get after some simplification,

$$\begin{aligned}\sum_{n=0}^{\infty} \frac{q^{n(n-1)}[q;q]_{3n}}{[q^3;q^3]_n[q^3;q^3]_{2n}} &= \frac{1}{[q;q]_\infty}\left[2+\sum_{n=1}^{\infty}(-)^n(1+q^n)(1+q^{2n})q^{n(5n-3)/2}\right] \\ &= \frac{1}{[q;q]_\infty}\left[\sum_{n=-\infty}^{\infty}(-)^n q^{n(5n-3)/2}+\sum_{n=-\infty}^{\infty}(-)^n q^{n(5n-1)/2}\right] \\ &= \frac{1}{[q,q^4;q^5]_\infty}+\frac{1}{[q^2,q^3;q^5]_\infty}.\end{aligned}$$

Thus, following identity appeals

$$\sum_{n=0}^{\infty} \frac{q^{n(n-1)}[q;q^3]_n[q^2;q^3]_n}{[q^3;q^3]_{2n}} = \frac{1}{[q,q^4;q^5]_\infty}+\frac{1}{[q^2,q^3;q^5]_\infty} \tag{3.27}$$

Following q-series identity results on comparing (3.19), (3.24) and (3.27) we get the following interesting q-series identity

$$\sum_{n=0}^{\infty} \frac{q^{n(n-1)}}{[q;q]_n} = \sum_{n=0}^{\infty} \frac{q^{n(n-1)}}{[q^2;q^2]_n[-q;q]_{2n}} = \sum_{n=0}^{\infty} \frac{q^{n(n-1)}[q;q^3]_n[q^2;q^3]_n}{[q^3;q^3]_{2n}}. \tag{3.28}$$

It may not be out of place to mention that many more results of the type evaluated in this paper can be evaluated by invoking other known Bailey's pairs. Presumably the results obtained in this paper can be given combinatorial interpretation.

Acknowledgement. This work has been done under Major Research Projects F. 31-83/2005 (S.R.) and F. No.32-164/2006(SR) sanctioned to the first two authors, respectively, by the University Grants Commission, Govt. of India, New Delhi, for which they wish to thank the authorities concerned. The authors are thankful to the Department of Science and Technology, Govt. of India, New Delhi, for support under major research projects sanctioned to them. Nidhi Sahni is a Junior Research fellow under the first author in his present project.

Authors express gratitude to the referee for his valuable comments which has improved the paper.

References

Andrews, G.E. (1970) : A polynomial identity which implies the Rogers-Ramanujan identities, *Scripta Math.*, **28**, 297.

Andrews, G.E. (1984) : Multiple series Rogers-Ramanujan type identities, *Pacific J. Math.*, **114**, 267.

Andrews, G.E. (1987) : Rogers-Ramanujan identities without Jacobi's triple product identity, *Rocky Mountain J. Math.*, **17**, 659.

Andrews, G.E. and **Berndt, B.C.** (2005) : Ramanujan's Lost Notebook Part I Springer, New York.

Bailey, W.N. (1949) : Identities of Rogers-Ramanujan type, *Proc. London Math. Soc.*, **50**(2), 1.

Berndt, B.C. (1994) : Ramanujan's Note Book Part IV, Springer, New York.

Carlitz, L. (1959) : Some formulas related to the Rogers-Ramanujan identities, *Annali di Matematica pura et applicata* (IV), vol. **XLVIII**, 247.

Garrett, K. (2006) : A determinant identity that implies Rogers-Ramanujan, *Electronic Journal of Combinatories*, **12**(1), 1.

Garrett, K.; Ismail, Mourad, E.H. and **Stanton, Dennis** (1999) : Variants of the Rogers-Ramanujan identities, *Advanes in Applied Mathematics*, **23**, 274.

Gasper, G. and **Rahman, M.** (1990) : Basic hypergeometric series, Cambridge University Press.

Slater, L.J. (1952) : Further identities of the Rogers-Ramanujan type, *Proc. London Math. Soc.*, **(2)54**, 147.

Stanton, Dennis (1989) : q-series and partitions, Ringer Verlag, New York.

Verma, A. (1980) : On identities of Rogers-Ramanujan type, *Indian Jour. Pure & Appld. Math.*, **11**(6), 770.

[1]DEPARTMENT OF MATHEMATICS
UNIVERSITY OF GORAKHPUR
GORAKHPUR 273009, U.P.
INDIA

[2]DEPARTMENT OF MATHEMATICS
T.D.P.G. COLLEGE
JAUNPUR 222002, U.P.
INDIA

CORRESPONDING AUTHOR : REMY Y. DENIS
E-MAIL : ddry@rediffmail.com