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On certain transformations involving basic hypergeometric series

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Abstract :

In this paper, making use of Bailey's Lemma and certain known summation formulae an attempt will be made to establish transformations involving terminating basic hypergeometric series. We shall deduce the transformations involving terminating and truncated series from our results.

Keywords and Phrases : Terminating basic hypergeometric function/series, Bailey's Lemma, truncated series and summation formula.

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1. INTRODUCTION, NOTATION AND DEFINITION :

Throughout this paper we shall adopt the following notation and defintion;

For any numbers a and q, real or complex and $\mid q \mid< 1, let$

$$[\alpha;q]_n \equiv [\alpha]_n = \left\{ \begin{array}{ll} (1-\alpha)(1-\alpha q)(1-\alpha q^2)........(1-\alpha q^{n-1}); & n>0 \\ 1 & ;n=0 \end{array} \right\} \tag{1.1}$$

Accordingly, we have

$$[\alpha;q]_\infty = \Pi_{r=0}^{\infty}(1-\alpha q^r).$$

Also,

$$[a_1, a_2, ..., a_r; q]_n \equiv [a_1; q]_n [a_2; q]_n ... [a_r; q]_n. \tag{1.2}$$

We define a basic hypergeometric series,

$${}_r\phi_s\left[\begin{array}{l} a_1, a_2, ..., a_r; q; z \\ b_1, b_2, ..., b_s; q^\lambda \end{array}\right] = \sum_{n=0}^{\infty} \frac{[a_1, a_2, ..., a_r; q]_n z^n q^{\lambda n(n+1)/2}}{[q, b_1, b_2, ..., b_s; q]_n}, (| z |< 1). \tag{1.3}$$

We, also, define a truncated series,

$${}_r\phi_s\left[\begin{array}{l} a_1, a_2, ..., a_r; q; z \\ b_1, b_2, ..., b_s; q^\lambda \end{array}\right]_N = \sum_{n=0}^{N} \frac{[a_1, a_2, ..., a_r; q]_n z^n q^{\lambda n(n+1)/2}}{[q, b_1, b_2, ..., b_s; q]_n},$$

Andrews [1] established the Bailey's Lemma in the following form,

If,

$$\beta_n = \sum_{r=0}^{n} \frac{\alpha_r}{[q; q]_{n-r}[aq; q]_{n+r}} \tag{1.4(a)}$$

then

$$\sum_{n=0}^{N} \frac{[\rho_1, \rho_2; q]_n (aq/\rho_1\rho_2)^n \alpha_n}{[aq/\rho_1, aq/\rho_2; q]_n [q; q]_{N-n} [aq; q]_{N+n}}$$

$$= \sum_{n=0}^{N} \frac{[\rho_1, \rho_2; q]_n [aq/\rho_1\rho_2; q]_{N-n} (aq/\rho_1\rho_2)^n \beta_n}{[q; q]_{N-n} [aq/\rho_1, aq/\rho_2; q]_N}, \tag{1.4(b)}$$

(Andrews [1 ;(2.3) and (2.4), p.270])

We shall need the following known results in our analysis,

$${}_4\phi_3\left[\begin{array}{l} a, q\sqrt{a}, -q\sqrt{a}, q^{-m}; q; -q^{-\frac{1}{2}+m} \\ \sqrt{a}, -\sqrt{a}, \quad aq^{1+m} \end{array}\right]$$

$$= \frac{[aq, -q^{-\frac{1}{2}}; q]_m}{2[\sqrt{(aq)}; q]_m[-q\sqrt{a}; q]_{m-1}} + \frac{[aq, -q^{-\frac{1}{2}}; q]_m}{2[-\sqrt{(aq)}; q]_m[q\sqrt{a}; q]_{m-1}}, \tag{1.5}$$

Verma and Jain [2 ;(4.1),p.76])

$${}_3\phi_2 \left[\begin{matrix} q, q\sqrt{a}, q^{-m}; q; -q^m \\ \sqrt{a}, \quad aq^{m+1} \end{matrix} \right]$$

$$= \frac{(1+\sqrt{a})[aq, -1; q]_m}{2[aq; q^2]_m} + \frac{(1-\sqrt{a})[aq; -1; q]_m}{2[\sqrt{a}, -q\sqrt{a}; q]_m}, \tag{1.6}$$

(Verma and Jain [2 ;(4.2), p.76])

$${}_2\phi_1 \left[\begin{matrix} a, q^{-m}; q; -q^{\frac{1}{2}+m} \\ aq^{m+1} \end{matrix} \right]$$

$$= \frac{(1+\sqrt{a})[aq, -\sqrt{q}; q]_m}{2[-\sqrt{a}q, q\sqrt{a}; q]_m} + \frac{(1-\sqrt{a})[aq, -\sqrt{q}; q]_m}{2[\sqrt{a}q, -q\sqrt{a}; q]_m}, \tag{1.7}$$

(Verma and Jain [2 ;(4.3), p.76])

$${}_4\phi_3 \left[\begin{matrix} a, q\sqrt{a}, -q\sqrt{a}, q^{-m}; q; -q^{\frac{1}{2}+m} \\ \sqrt{a}, -\sqrt{a}, \quad aq^{m+1} \end{matrix} \right] =$$

$$\frac{1}{2\sqrt{a}} \left\{ \frac{[aq, -q^{-\frac{1}{2}}; q]_m}{[\sqrt{(aq)}; q]_m[-q\sqrt{a}; q]_{m-1}} - \frac{[aq, -q^{-\frac{1}{2}}; q]_m}{[-\sqrt{aq}; q]_m[q\sqrt{a}; q]_{m-1}} \right\}, \tag{1.8}$$

(Verma and Jain [2 ;(4.5), p.77])

$${}_3\phi_2 \left[\begin{matrix} a, q\sqrt{a}, q^{-m}; q; -q^{m+1} \\ \sqrt{a}, \quad aq^{m+1} \end{matrix} \right] =$$

$$\frac{1+\sqrt{a}}{2\sqrt{a}}\frac{[aq,-1;q]_m}{[aq;q^2]_m}-\frac{1-\sqrt{a}}{2\sqrt{a}}\frac{[aq,-1;q]_m}{[\sqrt{(a)},-q\sqrt{a};q]_m} \tag{1.9}$$

(Verma and Jain [2 ;(4.6), p.77])

$${}_2\phi_1\left[\begin{matrix} a, q^{-m}; q; -q^{\frac{3}{2}+m} \\ aq^{m+1} \end{matrix}\right] =$$

$$\frac{1+\sqrt{a}}{2\sqrt{a}}\frac{[aq,-\sqrt{q};q]_m}{[-\sqrt{a}q,q\sqrt{a};q]_m}-\frac{1-\sqrt{a}}{2\sqrt{a}}\frac{[aq,-\sqrt{q};q]_m}{[\sqrt{a}q,-q\sqrt{a};q]_m} \tag{1.10}$$

(Verma and Jain [2 ;(4.7), p.77])

2. Transformations involving terminating series :

In this section we shall establish our main transformations. We shall also deduce certain transformations involving truncated series from our results.

If in (1.4a)we set

$$\alpha_n = \frac{1}{[q;q]_n}\left\{\frac{(1+\sqrt{a})[-q^{-\frac{1}{2}};q]_n}{2\sqrt{a}[\sqrt{a}q,-\sqrt{a};q]_n}-\frac{(1-\sqrt{a})[-q^{-\frac{1}{2}};q]_n}{2\sqrt{a}[-\sqrt{a}q,\sqrt{a};q]_n}\right\} \tag{2.9}$$

then, with the above values of α_n and the resulting value of β_n in (1.4(b)), we get

$$\frac{[aq/\rho_1,aq/\rho_2;q]_N}{[aq,aq/\rho_1\rho_2;q]_N}{}_6\phi_5\left[\begin{matrix} a,q\sqrt{a},-q\sqrt{a},\rho_1,\rho_2,q^{-N};q;-aq^{\frac{3}{2}+N}/\rho_1\rho_2 \\ \sqrt{a},-\sqrt{a},aq/\rho_1,aq/\rho_2,aq^{N+1} \end{matrix}\right]$$

$$=\frac{(1+\sqrt{a})}{2\sqrt{a}}{}_4\phi_3\left[\begin{matrix} \rho_1,\rho_2,-q^{-\frac{1}{2}},q^{-N};q;q \\ \sqrt{(aq)},-\sqrt{a},\rho_1\rho_2q^{-N}/a \end{matrix}\right]$$

$$-\frac{(1-\sqrt{a})}{2\sqrt{a}}{}_4\phi_3\left[\begin{matrix} \rho_1,\rho_2,-q^{-\frac{1}{2}},q^{-N},q;q \\ -\sqrt{(aq)},\sqrt{a},\rho_1\rho_2q^{-N}/a \end{matrix}\right] \tag{2.10}$$

Again, with

$$\alpha_r = \frac{q^{r(r-1)/2}[a, q\sqrt{a}; q]_r z^r}{[q, \sqrt{a}; q]r}$$

in (1.4(a)),we get

$$\beta_n = \frac{1}{[q, aq; q]_n} {}_3\phi_2 \left[\begin{array}{l} a, q\sqrt{a}, q^{-n}; q; -zq^n \\ \sqrt{a}, \quad aq^{n+1} \end{array} \right] \tag{2.11}$$

Now, taking $z = q$ in (2.11) and summing the ${}_3\phi_2$ with the help of (1.9) and substituting the resulting value of β_n and the above value of α_n, we get

$$\frac{[aq/\rho_1, aq/\rho_2; q]_N}{[aq, aq/\rho_1\rho_2; q]_N} {}_5\phi_4 \left[\begin{array}{l} a, q\sqrt{a}, , \rho_1, \rho_2, q^{-N}; q; -aq^{2+N}/\rho_1\rho_2 \\ \sqrt{a}, aq/\rho_1, aq/\rho_2, aq^{N+1} \end{array} \right]$$

$$= \frac{(1+\sqrt{a})}{2\sqrt{a}} {}_4\phi_3 \left[\begin{array}{l} \rho_1, \rho_2, -1, q^{-N}; q; q \\ \sqrt{(aq)}, -\sqrt{(aq)}, \rho_1\rho_2 q^{-N}/a \end{array} \right]$$

$$- \frac{(1-\sqrt{a})}{2\sqrt{a}} {}_4\phi_3 \left[\begin{array}{l} \rho_1, \rho_2, -1, q^{-N}; q; q \\ \sqrt{a}, -q\sqrt{a}, \rho_1\rho_2 q^{-N}/a \end{array} \right] \tag{2.12}$$

Further, if we set

$$\alpha_r = \frac{q^{r(r-1)/2}[a; q]_r z^r}{[q; q]_r}$$

in (1.4(a)) ,we get

$$\beta_n = \frac{1}{[q, aq; q]_n} {}_2\phi_1 \left[\begin{array}{l} a, q^{-n}; q; -zq^n \\ aq^{n+1} \end{array} \right] \tag{2.13}$$

Putting $z = q^{3/2}$ in (2.13) and using (1.10),to sum the ${}_2\phi_1$,we get,

$$\beta_n = \frac{1}{[q, aq; q]_n} \left\{ \frac{(1+\sqrt{a})[aq, -\sqrt{q}; q]_n}{2\sqrt{a}[-\sqrt{a}q, q\sqrt{a}; q]_n} - \frac{(1-\sqrt{a})[aq, -\sqrt{q}; q]_n}{2\sqrt{a}[\sqrt{a}q, -q\sqrt{a}; q]_n} \right\}$$

Substituting the above values of α_n and β_n in (1.14(b)), we get

$$\frac{[aq/\rho_1, aq/\rho_2; q]_N}{[aq, aq/\rho_1\rho_2; q]_N}\,{}_4\phi_3\left[\begin{matrix}\rho_1, \rho_2, a, q^{-N}; q; -aq^{\frac{5}{2}+N}/\rho_1\rho_2\\ aq/\rho_1, aq/\rho_2, aq^{N+1}\end{matrix}\right]$$

$$= \frac{1+\sqrt{a}}{2\sqrt{a}}\,{}_4\phi_3\left[\begin{matrix}\rho_1, \rho_2, -\sqrt{q}, q^{-N}; q; q\\ -\sqrt{(aq)}, q\sqrt{a}, \rho_1\rho_2 q^{-N}/a\end{matrix}\right]$$

$$-\frac{1-\sqrt{a}}{2\sqrt{a}}\,{}_4\phi_3\left[\begin{matrix}\rho_1, \rho_2, -\sqrt{q}, q^{-N}; q; q\\ \sqrt{(aq)}, -q\sqrt{a}, \rho_1\rho_2 q^{-N}/a\end{matrix}\right] \tag{2.14}$$

If we replace a by a/q in (1.4(a)) and (1.4(b)), we get the following form of Bailey's Lemma:

If,for n$\geq$ 0,

$$\beta_n = \sum_{r=0}^{n} \frac{\alpha_r}{[q;q]_{n-r}[a;q]_{n+r}} \tag{2.15}$$

then,

$$\sum_{n=0}^{N} \frac{[\rho_1, \rho_2; q]_n (a/\rho_1\rho_2)^n \alpha_n}{[a/\rho_1, a/\rho_2; q]_n [q;q]_{N-n}[a;q]_{N+n}}$$

$$= \sum_{n=0}^{N} \frac{[\rho_1, \rho_2; q]_n [a/\rho_1\rho_2; q]_{N-n} (a/\rho_1\rho_2)^n \beta_n}{[q;q]_{N-n}[a/\rho_1, a/\rho_2; q]_N} \tag{2.16}$$

We shall use the above two results, (2.15) and (2.16) to establish certain transformations involving terminating series.
If we set

$$\alpha_r = \frac{q^{r(r-1)/2}[a, q\sqrt{a}, -q\sqrt{a}; q]_r z^r}{[q, \sqrt{a}, -\sqrt{a}, ; q]_r}$$

in (2.15), we get

$$\beta_n = \frac{1}{[q,a;q]_n} {}_4\phi_3 \left[\begin{array}{l} a, q\sqrt{a}, -q\sqrt{a}, q^{-n}; q; -zq^n \\ \sqrt{a}, -\sqrt{a}, \quad aq^n \end{array} \right] \tag{2.17}$$

Now, taking $z = q^{-\frac{1}{2}}$ in the above and summing the ${}_4\phi_3$ with the help of the following known summation due to Verma and Jain[2;(4.15),p.80],

$${}_4\phi_3 \left[\begin{array}{l} a, q\sqrt{a}, -q\sqrt{a}, q^{-m}; q; -q^{-\frac{1}{2}+m} \\ \sqrt{a}, -\sqrt{a}, \quad aq^m \end{array} \right]$$

$$= \frac{[a, q\sqrt{a}, -q^{-\frac{1}{2}}; q]_m}{2[\sqrt{a}, \sqrt{(aq)}, -\sqrt{a}; q]_m} + \frac{[a, -q\sqrt{a}, -q^{-\frac{1}{2}}; q]_m}{2[-\sqrt{a}, -\sqrt{(aq)}, \sqrt{a}; q]_m}$$

we get

$$\beta_n = \frac{1}{2[q;q]_n} \left\{ \frac{[[q\sqrt{a}, -q^{-\frac{1}{2}}; q]_n}{[\sqrt{a}, \sqrt{(aq)}, -\sqrt{a}; q]_n} + \frac{[-q\sqrt{a}, -q^{-\frac{1}{2}}; q]_n}{[-\sqrt{a}, -\sqrt{(aq)}, \sqrt{a}; q]_n} \right\}$$

Substituting the above values of α_n and β_n in (2.16),we get

$$\frac{2[a/\rho_1, a/\rho_2; q]_N}{[a, a/\rho_1\rho_2; q]_N} {}_6\phi_5 \left[\begin{array}{l} a, q\sqrt{a}, -q\sqrt{a}, \rho_1, \rho_2, q^{-N}; q; -aq^{N-\frac{1}{2}}/\rho_1\rho_2 \\ a/\rho_1, a/\rho_2, \sqrt{a}, -\sqrt{a}, aq^N \end{array} \right]$$

$$= {}_5\phi_4 \left[\begin{array}{l} q\sqrt{a}, -q^{-\frac{1}{2}}, \rho_1, \rho_2, q^{-N}; q; q \\ \sqrt{a}, -\sqrt{a}, \sqrt{(aq)}, \rho_1\rho_2 q^{1-N}/a \end{array} \right]$$

$$+ {}_5\phi_4 \left[\begin{array}{l} -q\sqrt{a}, -q^{-\frac{1}{2}}, \rho_1, \rho_2, q^{-N}; q; q \\ \sqrt{a}, -\sqrt{a}, -\sqrt{(aq)}, \rho_1\rho_2 q^{1-N}/a \end{array} \right] \tag{2.18}$$

Next, if we put

$$\alpha_r = \frac{q^{r(r-1)/2}[a, q\sqrt{a}; q]_r z^r}{[q, \sqrt{a}; q]_r}$$

in (2.15) ,we get

$$\beta_n = \frac{1}{[q,a;q]_n}\,{}_3\phi_2\left[\begin{array}{l} a, q\sqrt{a}, q^{-n}; -zq^n \\ \sqrt{a}, \quad aq^n \end{array}\right]$$

Putting $z = 1$ in the above and summing the ${}_3\phi_2$ with the help of the following result due to Verma and Jain [2 ;(4.16), p.80],

$${}_3\phi_2\left[\begin{array}{l} a, q\sqrt{a}, q^{-m}; -q^m \\ \sqrt{a}, \quad aq^m \end{array}\right]$$

$$= \frac{[a, -1, q\sqrt{a}; q]_m}{2[\sqrt{a}, q\sqrt{(aq)}, -\sqrt{(aq)}; q]_m} + \frac{[a, -1; q]_m}{2[\sqrt{a}, -\sqrt{a}; q]_m}$$

we get

$$\beta_n = \frac{1}{2[q;q]_n}\left\{\frac{[q\sqrt{a}, -1; q]_n}{[\sqrt{a}, \sqrt{aq}, -\sqrt{aq}; q]} + \frac{[-1;q]_n}{[\sqrt{a}, -\sqrt{a}; q]_n}\right\}$$

Now, substituting the above values of α_n and β_n in (2.16), we get

$$\frac{2[a/\rho_1, a/\rho_2; q]_N}{[a, a/\rho_1\rho_2; q]_N}\,{}_5\phi_4\left[\begin{array}{l} a, q\sqrt{a}, \rho_1, \rho_2, q^{-N}; q; -aq^N/\rho_1\rho_2 \\ \sqrt{a}, a/\rho_1, a/\rho_2, aq^N \end{array}\right]$$

$$= {}_5\phi_4\left[\begin{array}{l} q\sqrt{a}, \rho_1, \rho_2, , -1, q^{-N}; q; q \\ \sqrt{a}, \sqrt{(aq)}, -\sqrt{(aq)}, \rho_1\rho_2 q^{1-N}/a \end{array}\right]$$

$$+ {}_4\phi_3\left[\begin{array}{l} \rho_1, \rho_2, , -1, q^{-N}; q; q \\ \sqrt{a}, -\sqrt{(a)}, \rho_1\rho_2 q^{1-N}/a \end{array}\right] \tag{2.19}$$

Further, setting

$$\alpha_r = \frac{q^{r^2/2}[a;q]_r}{[q;q]_r}$$

in (2.15) and making use of yet another known result due to Verma and Jain [2 ;(4.17),p.80],namely,

$${}_2\phi_1\left[\begin{array}{l} a, q^{-m}; q; -q^{m+\frac{1}{2}} \\ aq^m \end{array}\right] = \frac{[a, -\sqrt{q}; q]_m}{2[-\sqrt{(aq)}, \sqrt{a}; q]_m} + \frac{[a, -\sqrt{q}; q]_m}{2[\sqrt{(aq)}, -\sqrt{a}; q]_m}$$

we get

$$\beta_n = \frac{1}{2[q;q]_n}\left\{\frac{[-\sqrt{q};q]_n}{[\sqrt{a},-\sqrt{(aq)};q]_n} + \frac{[-\sqrt{q};q]_n}{[-\sqrt{a},\sqrt{(aq)};q]_n}\right\}$$

Substituting the above values of α_n and β_n in (2.16),we get

$$\frac{2[a/\rho_1,a/\rho_2;q]_N}{[a,a/\rho_1\rho_2;q]_N}{}_4\phi_3\left[\begin{matrix} a,\rho_1,\rho_2,q^{-N};q;-aq^{N+\frac{1}{2}}/\rho_1\rho_2 \\ a/\rho_1,a/\rho_2,aq^N \end{matrix}\right]$$

$$= {}_4\phi_3\left[\begin{matrix} \rho_1,\rho_2,-\sqrt{q},q^{-N};q;q \\ \sqrt{a},-\sqrt{(aq)},\rho_1\rho_2q^{1-N}/a \end{matrix}\right]$$

$$+{}_4\phi_3\left[\begin{matrix} \rho_1,\rho_2,-\sqrt{q},q^{-N};q;q \\ -\sqrt{a},\sqrt{(aq)},\rho_1\rho_2q^{1-N}/a \end{matrix}\right] \tag{2.20}$$

Now,if we replace a by aq in (1.4(a)) and (1.4(b)),we get:

If ,for $n \geq 0$,

$$\beta_n = \sum_{r=0}^{n}\frac{\alpha_r}{[q;q]_{n-r}[aq^2;q]_{n+r}} \tag{2.21}$$

then

$$\sum_{n=0}^{N}\frac{[\rho_1,\rho_2;q]_n(aq^2/\rho_1\rho_2)^n\alpha_n}{[aq^2/\rho_1,aq^2/\rho_2;q]_n[q;q]_{N-n}[aq^2;q]_{N+n}}$$
$$= \sum_{n=0}^{N}\frac{[\rho_1,\rho_2;q]_n[aq^2/\rho_1\rho_2;q]_{N-n}(aq^2/\rho_1\rho_2)^n\beta_n}{[q;q]_{N-n}[aq^2/\rho_1,aq^2/\rho_2;q]_N} \tag{2.22}$$

Now,if we put

$$\alpha_r = \frac{q^{r(r+1)/2}[a,q\sqrt{a};q]_r}{[q,\sqrt{a};q]_r}$$

and make use of the following known summation due to Verma and Jain [2 ;(4.8),p.79],

$$ {}_3\phi_2\left[\begin{array}{l} a, q\sqrt{a}, q^{-m}; q; -q^{m+1} \\ \sqrt{a},\ \ aq^{m+2} \end{array}\right] $$

$$ = \frac{[a;q]_{m+2}[q\sqrt{a},-1;q]_{m+1}[q;q]_m}{2\sqrt{a}[q,-\sqrt{(aq)},\sqrt{a},\sqrt{a}q;q]_{m+1}} - \frac{[a;q]_{m+2}[-1;q]_{m+1}[q;q]_m}{2\sqrt{a}[\sqrt{a},-\sqrt{a},q;q]_{m+1}} $$

to calculate the value of β_n and then substitute this value of β_n and the above value of α_n in(2.22),we get

$$ \frac{[aq^2/\rho_1, aq^2/\rho_2;q]_N}{[aq^2, aq^2/\rho_1\rho_2]_N}{}_5\phi_4\left[\begin{array}{l} a, q\sqrt{a}, rho_1, \rho_2, q^{-N}; q; -aq^{3+N}/\rho_1\rho_2 \\ \sqrt{a}, aq^2/\rho_1, aq^2/\rho_2, aq^{N+2} \end{array}\right] $$

$$ = \frac{(1-q\sqrt{a})(1+\sqrt{a})}{2\sqrt{a}(1-q)}{}_6\phi_5\left[\begin{array}{l} \rho_1, \rho_2, q, -q, q^2\sqrt{a}, q^{-N}; q; q \\ \rho_1\rho_2 q^{-N-1}/a, q^2, q\sqrt{a}, q\sqrt{(aq)}, -q\sqrt{(aq)} \end{array}\right] $$

$$ -\frac{(1-aq)}{2\sqrt{a}(1-q))}{}_5\phi_4\left[\begin{array}{l} \rho_1, \rho_2, q, -q, q^{-N}; q; q \\ \rho_1\rho_2 q^{-N-1}/a, q^2, -q\sqrt{a}, q\sqrt{a} \end{array}\right] \tag{2.23} $$

Further, if we put

$$ \alpha_r = \frac{q^{r^2/2}[a, q\sqrt{a}, -\sqrt{a}, q]_r}{[q, \sqrt{a}, -\sqrt{a}; q]_r} $$

in (2.21) and use the following known summation due to Verma and Jain [2;(4.9),p.78],

$$ {}_4\phi_3\left[\begin{array}{l} a, q\sqrt{a}, -q\sqrt{a}, q^{-m}; q; -q^{m+\frac{1}{2}} \\ \sqrt{a}, -\sqrt{a}, aq^{2+m} \end{array}\right] $$

$$ = \frac{[a;q]_{m+2}[q\sqrt{a}, -q^{-\frac{1}{2}}; q]_{m+1}[q;q]_m}{2\sqrt{a}[q, \sqrt{a}, -\sqrt{a}, \sqrt{(aq)}]_{m+1}} $$

$$ -\frac{[a;q]_{m+2}[-q\sqrt{a}, -q^{-\frac{1}{2}}; q]_{m+1}[q;q]_m}{2\sqrt{a}[q, \sqrt{a}, -\sqrt{a}, -\sqrt{(aq)}]_{m+1}} \tag{2.24} $$

for the evaluation of β_n,we get, after proper substitution for α_n and β_n in (2.22)

$$\frac{[aq^2/\rho_1, aq^2/\rho_2]_N}{[aq^2, aq^2/\rho_1\rho_2; q]_N} {}_6\phi_5 \left[\begin{matrix} \rho_1, \rho_2, a, q\sqrt{a}, -q\sqrt{a}, q^{-N}; q; -aq^{\frac{5}{2}+N}/\rho_1\rho_2 \\ aq^2/\rho_1, aq^2/\rho_2, \sqrt{a}, -\sqrt{a}, \quad aq^{2+N} \end{matrix} \right]$$

$$= \frac{(1+\sqrt{(aq)})(1+q\sqrt{a})}{2\sqrt{(aq)}(1-\sqrt{q})} {}_6\phi_5 \left[\begin{matrix} \rho_1, \rho_2, -q^2\sqrt{a}, -q^{\frac{1}{2}}, q, q^{-N}; q; q \\ \rho_1\rho_2 q^{-N-1}/a, q^2, q\sqrt{a}, -q\sqrt{a}, q\sqrt{(aq)} \end{matrix} \right]$$

$$- \frac{(1-\sqrt{(aq)})(1+q\sqrt{a})}{2\sqrt{(aq)}(1-\sqrt{q})} {}_6\phi_5 \left[\begin{matrix} \rho_1, \rho_2, -q^2\sqrt{a}, -q^{\frac{1}{2}}, q, q^{-N}; q; q \\ \rho_1\rho_2 q^{-N-1}/a, q^2, q\sqrt{a}, -q\sqrt{a}, -q\sqrt{(aq)} \end{matrix} \right] \quad (2.25)$$

Finally, if we set

$$\alpha_r = \frac{q^{r(r+2)/2}[a;q]_r}{[q;q]_r}$$

in (2.21) and use the following known summation due to Verma and Jain [2;(4.11),p.79],namely,

$${}_2\phi_1 \left[\begin{matrix} a, q^{-m}; q; -q^{m+\frac{3}{2}} \\ aq^{2+m} \end{matrix} \right]$$

$$= \frac{[a;q]_{m+2}[-\sqrt{q};q]_{m+1}[q;q]_m}{2\sqrt{a}[q, \sqrt{a}, -\sqrt{(aq)};q]_{m+1}} - \frac{[a;q]_{m+2}[-\sqrt{q};q]_{m+1}[q;q]_m}{2\sqrt{a}[q, -\sqrt{a}, \sqrt{(aq)};q]_{m+1}}$$

to evaluate β_n, we get

$$\beta_n = \frac{1}{[q;q]_n} \left\{ \frac{(1-a)(1-aq)(1+\sqrt{q})[q, -q^{3/2};q]_n}{2\sqrt{a}(1-\sqrt{a})(1-q)(1+\sqrt{(aq)})[q\sqrt{a}, q^2, -q\sqrt{(aq)};q]_n} \right.$$
$$\left. - \frac{(1-a)(1-aq)(1+\sqrt{q})[q, -q^{3/2};q]_n}{2\sqrt{a}(1-\sqrt{(aq)})(1+\sqrt{a})(1-q)[q\sqrt{(aq)}, -q\sqrt{a}, q^2;q]_n} \right\}$$

Substituting the above values of α_n and β_n in (2.22), we get

$$\frac{[aq^2/\rho_1, aq^2/\rho_2]_N}{[aq^2, aq^2/\rho_1\rho_2; q]_N}\ {}_4\phi_3\left[\begin{array}{l} \rho_1, \rho_2, a, q^{-N}; q; -aq^{\frac{7}{2}+N}/\rho_1\rho_2 \\ aq^2/\rho_1, aq^2/\rho_2, \quad aq^{2+N} \end{array}\right]$$

$$= \frac{(1-a)(1-aq)(1+\sqrt{q})}{2\sqrt{a}(1-q)(1+\sqrt{(aq)})(1-\sqrt{a})}\,{}_5\phi_4\left[\begin{array}{l} \rho_1, \rho_2, q^{-N}, q, -q^{3/2}; q; q \\ \rho_1\rho_2 q^{-N-1}/a, q^2, q\sqrt{a}, -q\sqrt{(aq)} \end{array}\right] -$$

$$-\frac{(1-a)(1-aq)(1+\sqrt{q})}{2\sqrt{a}(1-q)(1-\sqrt{(aq)})(1+\sqrt{a})}\,{}_5\phi_4\left[\begin{array}{l} \rho_1, \rho_2, q^{-N}, q, -q^{3/2}; q; q \\ \rho_1\rho_2 q^{-N-1}/a, q^2, -q\sqrt{a}, q\sqrt{(aq)} \end{array}\right] \quad (2.26)$$

The transformations investigated here involve terminating series on both sides. From the above results we can deduce the transformations in which both the series on the right are truncated. We sight the following result.

If we let $a \to \rho_1\rho_2$ in (2.3), we get

$$\frac{[q\rho_1, q\rho_2; q]_N}{[q\rho_1\rho_2, q; q]_N}\,{}_6\phi_5\left[\begin{array}{l} \rho_1\rho_2, q\sqrt{(\rho_1\rho_2)}, -q\sqrt{(\rho_1\rho_2)}, \rho_1, \rho_2, q^{-N}; q; -q^{N+\frac{1}{2}} \\ \sqrt{(\rho_1\rho_2)}, -\sqrt{(\rho_1\rho_2)}, q\rho_1, q\rho_2, \rho_1\rho_2 q^{N+1} \end{array}\right]$$

$$= \frac{1+\sqrt{(\rho_1\rho_2)}}{2}\,{}_3\phi_2\left[\begin{array}{l} \rho_1, \rho_2, -q^{-\frac{1}{2}}; q; q \\ \sqrt{(\rho_1\rho_2 q)}, -\sqrt{(\rho_1\rho_2)}, \end{array}\right]_N$$

$$+\frac{1-\sqrt{(\rho_1\rho_2)}}{2}\,{}_3\phi_2\left[\begin{array}{l} \rho_1, \rho_2, -q^{-\frac{1}{2}}; q; q \\ -\sqrt{(\rho_1\rho_2 q)}, \sqrt{(\rho_1\rho_2} \end{array}\right]_N \quad (2.27)$$

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